How did I end up here and how are inverse problems related?

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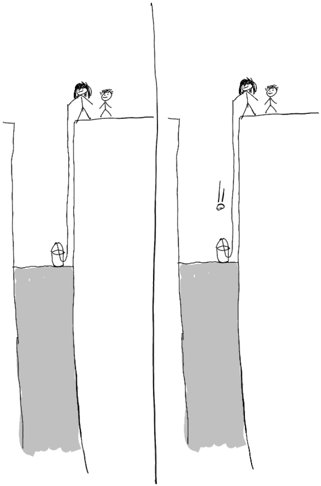
Botswana

[Picture of 5-year old Emilia, her brother, mother and the latter's friends all having lunch on the terrace in Botswana.]

Beginning of scientific career

[Picture of 5-year old Emilia, backlit, inside a shed, building something out of wood.]



France and the magic of maths

[8-year old Emilia thinking while playing chess against her little brother who is guided by Mum.]

“Casting out nines”

$$312 \times 852 \neq 266824$$

because $(3+1+2)\times(8+5+2) \to 6\times 6 \to 36 \to 0$ but $2+6+6+8+2+4 \to 1$

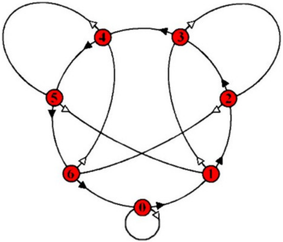


Image credits: David Wilson

Recent: (mod 7) is interesting

$324 \mod 7 = ?$

$$0 \xrightarrow{3B} 3 \xrightarrow[1W]{} 2 \xrightarrow{2B} 4 \xrightarrow[1W]{} 5 \xrightarrow{4B} 2$$

so $324 \equiv 2 \pmod 7$

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Teenage years, high school suffering and escapism

After high school: University of Helsinki, degrees, inverse problems research group, PhD.

After PhD: Resolvent estimates à la Hörmander

Let $(\Delta + k^2)u = f$, then

- Agmon (1975), $\delta > \frac{1}{2}$

$$\|(1+|x|^2)^{-\delta/2}u\|_{L^2(\mathbb{R}^n)} \leq \frac{C}{k}\|(1+|x|^2)^{\delta/2}f\|_{L^2(R^n)}$$

- Agmon–Hörmander (1976), $A_j = \{2^{j-1} < |x| < 2^{j+1}\}$, $A_0 = \{|x| < 2\}$

$$\sup_{j\geq 0} \frac{1}{2^{j/2}}\|u\|_{L^2(A_j)} \leq \frac{C}{k}\sum_{j=0}^{\infty} 2^{j/2}\|f\|_{L^2(A_j)}$$

- Kenig–Ruiz–Sogge (1987), $\frac{1}{q_1} + \frac{1}{q_2} = 1$, $\frac{2}{n+1} \leq \frac{1}{q_1} - \frac{1}{q_2} \leq \frac{2}{n}$

$$\|u\|_{L^{q_2}(\mathbb{R}^n)} \leq Ck^{n(\frac{1}{q_1}-\frac{1}{q_2})-2}\|f\|_{L^{q_1}(\mathbb{R}^n)}$$

None of the above are satisfactory from a physical point of view: dilation, rotation, translation, behaviour w.r.t. wavenumber. . .

Physical estimates for scattering theory

Theorem (John Sylvester 2013 or earlier)

If $\operatorname{supp} f \subset D_1$ *is bounded then* $(\Delta + k^2)u = f$ *has a scattering solution* u*. It satisfies*

$$\|u\|_{L^2(D_2)} \leq C \frac{\sqrt{\operatorname{diam}(D_1)}\sqrt{\operatorname{diam}(D_2)}}{k} \|f\|_{L^2(D_1)}$$

for any bounded D_2*.*

Corollary (Generalised Agmon–Hörmander estimates)

Having split $f = \sum_j f_j$, $\operatorname{supp} f_j \subset D_j$ *(not necessarily annuli!), we have*

$$\sup_{D \subset \mathbb{R}^n} \frac{1}{\sqrt{\operatorname{diam}(D)}} \|u\|_{L^2(D)} \leq \frac{C}{k} \sum_j \sqrt{\operatorname{diam}(D_j)} \|f_j\|_{L^2(D_j)}$$

With John Sylvester, we wanted estimates that have physical symmetry but are still for a general $P(D)$ and not just $\Delta + k^2$.

Theorem (Blåsten & Sylvester 2017)

Let P be an admissible polynomial, $\operatorname{supp} f \subset D_1$ bounded. Then $P(D)u = f$ has a solution u that satisfies

$$\|u\|_{L^2(D_2)} \leq C \frac{\sqrt{\mathrm{diam}(D_1)}\sqrt{\mathrm{diam}(D_2)}}{k} \|f\|_{L^2(D_1)}$$

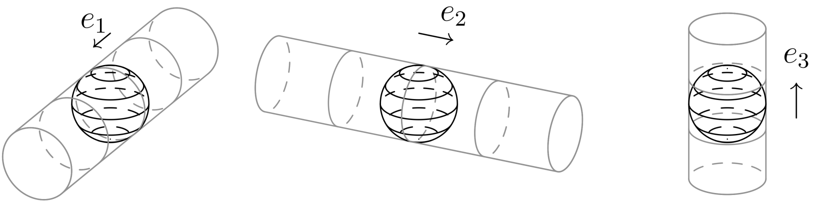
for any bounded D_2.

Admissible means it's constant coefficient and either

- real, of second order, and with no real double characteristics, or
- of N-th order, whose symbol satisfies certain inequalities related to the algebraic varieties (where $\theta \in \mathbb{S}^{n-1}$)

$$\mathcal{B}(\theta) = \left\{ \xi \in \mathbb{R}^n \;\middle|\; \text{the line } \tau \mapsto \xi + \tau\theta \text{ is tangent to } P^{-1}(0) \right\}$$

Proof idea: Fourier transform on non-tangential lines



We need to show that there are directions $\theta_1, \ldots, \theta_m$ such that

1. Every $\xi \in \mathbb{R}^n$ has some θ_j such that $\xi \notin \mathcal{B}(\theta_j)$, and
2. that there are estimates tying a partition of unity to the tangent varieties $\mathcal{B}(\theta_1), \ldots, \mathcal{B}(\theta_m)$

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Coming out to myself

Photo by myself, near Tsim Sha Tsui Star Ferry pier, Hong Kong 2018

Was transition worth it, a-posteriori?

My privilege got reduced:

white western man $\Longrightarrow$ one of the most hated minorities

Instead I gained:

- life enjoyment
- seeing worthy goals in the future
- getting to live like I want, instead of constantly thinking "what would others think about..."
- having a very unique perspective to life, space and time!

Queer time

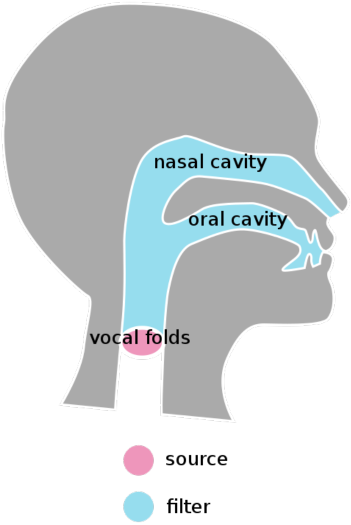
"Queer time for me is the dark nightclub, the perverse turn away from the narrative coherence of adolescence – early adulthood – marriage – reproduction – child rearing – retirement – death, the embrace of late childhood in place of early adulthood or immaturity in place of responsibility."

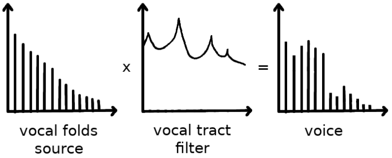
— J. Halberstam

"The possibility that somebody who's literally younger than you might be a senior trans person because they've been out for longer."

— @PhilosophyTube

Source-filter model, hearing gender from voice





In the vocal tract, $\Box_A u = 0$,

$$\Box_A u = \partial_t^2 u - \frac{1}{A(x)} \partial_x \big(A(x) \partial_x u \big)$$

If source and filter are unknown, determining them from speech is called *glottal inverse filtering*. An open problem.

Noise measurement

. . . an easier interesting problem to tackle

$$\begin{aligned} \Box_A u(x,t) &= 0, && x > 0,\ t \in \mathbb{R} \\ \partial_x u(0,t) &= \chi(t)\omega(t), && t \in \mathbb{R} \\ u(x,t) &= 0, && x > 0, t < 0. \end{aligned}$$

χ smooth ramp-up function, $\omega(t)$ realisation of white noise,
$0 < c < A(x) < C < \infty$ smooth cross-sectional area function,
measurement $\Lambda_{A,\omega}(t) = u(0,t)$.

Theorem (B., Helin, Kujanpää, Oksanen, Railo (submitted))

$A_1 \neq A_2$ iff there is a measurable $U \subset \mathcal{S}'(\mathbb{R})$ with $\mathbb{P}(U) = 1$ such that $\Lambda_{A_1,\omega} \neq \Lambda_{A_2,\omega}$ for every $\omega \in U$.

Common language formulation:

Theorem (B., Helin, Kujanpää, Oksanen, Railo (submitted))

If external white noise is applied at the lips, then the rough shape of the vocal tract can be determined by the echo almost surely.

Proof idea

1. The correlation operator C_T

$$\langle C_T(\varphi), \psi \rangle := \frac{1}{T} \int_0^T \langle \tau_s^* \Lambda_{A,\omega}, \varphi \rangle \langle \tau_s^* \omega, \psi \rangle \, \mathrm{d}s$$

   measures the average correlation between the input and output in different time intervals using the test functions φ, ψ.

2. Passing to the adjoint of Λ, we get expressions of the form

$$\langle \omega, f \rangle \langle \omega, g \rangle$$

   with non-stochastic f, g. Its expected value is $\langle f, g \rangle$.

3. Ergodic properties of Gaussian white noise allow us to change $\lim_{T\to\infty} \frac{1}{T} \int_0^T \ldots \mathrm{d}s$ to $\mathbb{E}(\ldots)$.
4. To do this we require a decay over time estimate for $\Lambda_{A,\omega}(t)$.
5. Combining everything yields the proof.

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Thank you!